Monodispersion of the Investigated Flow in the Sorting Unit of a Particle Size Optical and Electrical Analyser

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Abstract

The use of a compact cylindrical piezoelectric transducer in the jet dispersion unit of the sorting system of an optical or electrical particle size analyser is discussed. Solving two equations for the shift distribution in a piezoelectric transducer with a running fluid chamber, important expressions for the vibrational velocity of acoustic terminal, mechanical impedance, force factor, resonance frequency of dispersion unit and length of the stable part of the jet are obtained. The practical application of this technique is illustrated by the calculation of design parameters for an industrial dispersion unit used in an H-50 cytophotometer (Ortho Instruments, USA). Good agreement between theoretical and experimental results is shown.

1 Introduction

It is known [1-3] that the automatic sorting of investigated particles according to one or more particle parameters, measured (with a rate of up 5000 particles per second) in optical and electrical running fluid analysers, considerably increases the precision and the range of measurements. This is due to the possibility of carrying out additional investigations on particles, preliminarily sorted in the analyser on any basis, by other methods (microscopic, X-ray, etc.) [3]. Sorting of particles is very important for some technological processes. The sorting of the particles is effected in an electric field as a result of breaking down the jet, including the suspension of the particles passing one by one through the measuring zone of the analyser's special running fluid chamber [4], into monodisperse drops, and a charge is given by the synchronization system to each of these drops [5] that is proportional to the earlier measured parameter of a particle presents in the drop. As a rule, monodispersion of the jet takes place under the action of harmonic vibrations, produced by a piezoelectric transducer.

The known structures of composite piezoelectric transducers [1,2,6-11], which consist of sets of piezoelectric disks and metal straps, have the essential disadvantage of the positioning of the running fluid chamber next to the transducer. As a result, the size and weight of the dispersion unit are increased and the distance between the feeding system of investigated suspension and running fluid chamber is increased. The latter effect has a negative result on the accuracy of measuring and sorting the particles, because the sedimentation factor increases (especially for large-sized fractions) and breaks the principle of measuring each particle separately. These disadvantages are absent in a dispersion unit with a cylindrical piezoelectric transducer with the dispersion unit, as for example in the H-50 cytophotometer (Ortho Instruments, USA) [3]. However, at present there is little information about design problems of particular elements of the optimum sorting unit for particle size analysers.

The object of this work was to develop an engineering technique for the simulation of the design parameters of the sorting units of optical and electrical particle counters.

2 Design Technique

Figure 1 shows the dispersion unit (DU) with a cylindrical piezoelectric transducer (1), polarized in the radial direction and attached rigidly by the edge to the positioning table (8) of the analyser. Longitudinal vibrations, generated because of inverse

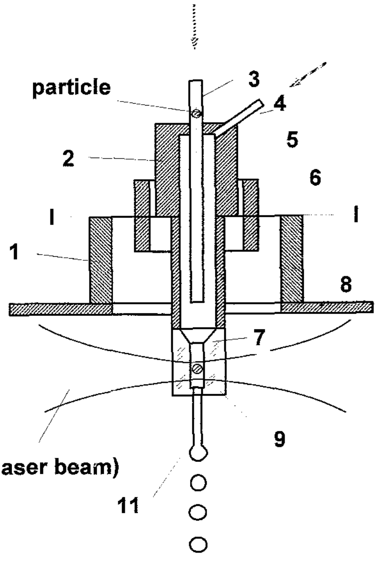

Fig. 1: Generalized diagram of the dispersion unit and running-type chamber: 1 =piezoelectric transducer; 2–4 = upper part of running chamber (2 = capillary; 3 = pipe; 4 = inlet for injecting liquid of sheath); 5, 6 = upper and lower parts of ring; 7 = lower part of running chamber; 8 = position table; 9 = quartz dish; 10 = laser beam; 11 = jet of suspension particles.

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piezoelectricity when an electrical voltage Eexp(jωt) with a frequency $f = \omega/2\pi$ is applied to the inner and outer metallic surfaces of the cylindrical piezoelectric transducer, are transmitted to the running fluid chamber (RFC), divided into an upper part 2 (RFC2), situated above the plane (I) of the chamber rigid mount with the operational edge of the piezoelectric transducer and a lower part 7 (RFC7), situated under the plane (I). The ring, conditionally divided by the RFC principle into upper (5) and lower (6) parts, provides easy dismantling and mounting of the RFC in the dispersion unit for replacement and maintenance purposes.

A technique for the design of the RFC was presented earlier [4]. The flow of suspension including the investigated particles, which are fed into the RFC through a needle (3), is focused in the measuring area of the laser beam (10) in a quartz dish (9) thanks to its hydrodynamic compression by the "sheath" (distilled water) being fed to the RFC through the coupling (4). Other structures of the RFC are possible, including ones with two "sheaths" [4] and with an electrical sensor instead of the dish (9). The RFC7 free edge, transmitting vibrations to the flowing out jet (11) of suspension particles, is the DU acoustic terminal for all structures of running fluid chambers.

Let the origin of the OX axis, directed to the side opposite the flowing out jet, coincide with the free edge of RFC7, and with the following structural constraint imposed on the elements 1, 2 and 5-7 of dispersion unit: $z_{oi} = \rho_i c_i S_i$ is constant along the length of each ith (i = 1, 2, 5-7) element, where ρ_i and S_i are the material density and cross-section area of the ith element and c_i is the velocity of sound propagation in the ith element. This constraint is fulfilled without damaging the provision of hydrodynamic focusing of the flow of the investigated particles in the measuring area. Then, from joint consideration of well-known [11] equations, namely the equation of the shift distribution u_7 in RFC7 for the case of an alternating stimulating force $F_7 \exp(j\omega t)$ action on the free edge (x = 0) with shortened electrodes of piezoelectric transducer (1)

$$u_7 = \frac{F_7}{\omega z_{07}} \, \frac{\cos\frac{\omega}{c_7}(l_7 - x) + j\frac{z_{\Sigma 7}}{z_{07}}\sin\frac{\omega}{c_7}(l_7 - x)}{\left(\mathrm{tg}\beta_7 - j\frac{z_{\Sigma 7}}{z_{07}}\right)\cos\beta_7} \tag{1}$$

and also the equation of the shift distribution u_1 in piezoelectric transducer (1), when an electrical voltage Eexp(jωt) is applied to its metallic surfaces

$$u_1 = -\frac{e_{32}S_1E}{\omega(r_2 - r_1)} \, \frac{\sin\frac{\omega}{c_1}(l_1 - l_7 + x)}{z_{07}\mathrm{tg}\beta_7 - jz_{\Sigma 7}}\,, \quad l_1 \le x \le l_7 \tag{2}$$

we obtain the following equations:

$$v(0) = j\omega u_7(x = 0) = -j\frac{F_7}{z_{07}} \, \frac{1 + j\frac{z_{\Sigma 7}}{z_{07}}\mathrm{tg}\beta_7}{\mathrm{tg}\beta_7 - j\frac{z_{\Sigma 7}}{z_{07}}} \tag{3}$$

is the vibrational velocity of the acoustic terminal of DU, $z = F_7/v(0)$ is the mechanical impedance of DU,

$$A = e_{32}\pi\,(r_2 - r_1)\cos\beta_7 \tag{4}$$

is the force factor [11], which was obtained from the condition $u_1 = u_7\,|_{x=l_7}$, and

$$\mathrm{tg}\beta_7 - j\frac{z_{\Sigma 7}}{z_{07}} = 0 \tag{5}$$

is the frequency equation determining the condition of resonance for DU (x = 0).

In the above equations, $z_{\Sigma 7} = z_1 + z_2 + z_5 + z_6$ is the impedance, on which RFC7 is loaded, z_1, z_2, z_5, and z_6 are the impedances of the piezoelectric transducer, RFC2 and ring 5,6 respectively, defined on the plane I side by the equations $z_1 = -jz_{01}\,\mathrm{ctg}\beta_1$, $z_2 = jz_{02}\,\mathrm{tg}\beta_1$, $z_5 = jz_{05}\mathrm{tg}\beta_5$ and $z_6 = jz_{06}\mathrm{tg}\beta_6$, $\beta_i = \omega l_i/c_i$ (i = 1, 2, 5-7) [11], l_i are the lengths of the respective ith DU elements, r_2 and r_1 are the outer and inner radii of the piezoelectric transducer and e_{32} is the piezoelectric constant of voltage for transducer (1).

In addition to Eq. (5), stating the relationship between structural parameters of DU and its resonant frequency, for the evaluation of the jet dispersion efficiency it is necessary to know the amplitude of the vibrational velocity of the DU acoustic terminal on the resonant frequency

$$v_r(0) = \frac{AE}{R_M}\,, \tag{6}$$

where $R_M = R + R_A$ and R and R_A are the active mechanical resistance of DU and the resistance of internal mechanical losses and active resistance of radiation, respectively. The resistance of mechanical losses R_M, with virtually no influence on the DU's resonant frequency [11], defines the value of the vibrational velocity (6) and therefore the sharpness of the resonance, characterized by the Q-factor of the dispersion unit:

$$Q = \frac{\omega_r m}{R_M} = \frac{\omega_r}{|\omega_2 - \omega_1|}\,, \tag{7}$$

where m is the equivalent DU mass, ω_2 and ω_1 are the frequencies at which the vibrational velocity is halved compared with its maximum value and ω_r is the resonant circular frequency. Taking into account Eq. (5), the equivalent DU mass on the resonance is defined by the equation [11]

$$m = -2j\frac{\partial z}{\partial \omega}\Big|_{\mathrm{tg}\beta_7 = j\frac{z_{\Sigma 7}}{z_{07}}} \tag{8}$$

and equals

$$m = \frac{1}{2}\left(M_7 + M_\Sigma \cos^2\beta_7\right), \tag{9}$$

where

$$M_\Sigma = \frac{M_1}{\sin^2\beta_1} + \frac{M_2}{\cos^2\beta_2} + \frac{M_5}{\cos^2\beta_5} + \frac{M_6}{\cos^2\beta_6}\,; \tag{10}$$

M_i (i = 1, 2, 5-7) are the mechanical masses of the respective ith DU's elements. Having taken into account Eqs. (4) and (6)-(9), we can write the final expression for the amplitude of the vibrational velocity of DU's acoustic terminal on the resonance

$$v_r(0) = 2\,\frac{EQe_{32}\pi(r_2 + r_1)\cos\beta_7}{\omega_r(M_7 + M_\Sigma\cos^2\beta_7)}\,, \tag{11}$$

where Q is expected or defined on the DU breadboard system's Q-factor. Examination of Eq. (11) for the extremum shows that vibrational velocity has a maximum value for DU structures, fulfilling the condition

$$\cos\beta_7 = \pm\sqrt{\frac{M_7}{M_\Sigma}}\,. \tag{12}$$

The distance l from RFC7's edge, on which the jet flowing out with velocity V_j is broken down into equal-sized drops under the action of vibrations, is an important characteristic of DU. During the period $T = 2\,\pi/\omega$ of harmonic vibrations of RFC7's edge with amplitude $v_r(0)/\omega$, fluid of elementary volume $\pi(r_0\varepsilon)^2\,V_j$ flows out of the chamber opening with radius r_0, where ε is contraction factor of the opening. The elementary volume will contract or expand along the length of value $v_r(0)/\omega$ alternately with frequency 1/T. Inasmuch as liquid is incompressible (the value of the elementary volume is constant), its radius will change with the denoted frequency. Then the amplitude of the periodical disturbances arising on the jet's surface

$$\alpha_0 = r_0\,\varepsilon\left[1 - \frac{1}{\sqrt{1+\dfrac{v_r(0)}{2\pi V_j}}}\right], \tag{13}$$

according to Rayleigh theory [12], grows exponentially. The condition on the jet's break-up is

$$\alpha_0 \exp\left(\frac{q_1 l}{V_j}\right) = r_0\varepsilon\,, \tag{14}$$

where $q_1 = q(\mu)\,[T_t/\rho\,(r_0^3\,\varepsilon^3)]^{1/2}$, T_t and ρ are the surface tension and the density of liquid, q(μ) is a factor defined in [12] and $\mu = r_0\,\varepsilon\,\omega/V_j$ is the initial value for the definition of the required distance l from RFC edge to the point of the jet's break-up

$$l = \frac{V_j}{q_1}\left(1 - \ln\frac{1}{1-\dfrac{1}{\sqrt{1+\dfrac{v_r(0)}{2\,\pi\,V_j}}}}\right). \tag{15}$$

According to Rayleigh's data and later investigations, the maximum value $q(\mu) = 0.343$ is achieved when $\mu = 0.696$. Hence it follows the condition for the optimum selection of the periodical jet disturbance frequency (DU's resonant frequency) is

$$f_r = \frac{V_j}{4{,}5\,(2 r_0\,\varepsilon)}\,. \tag{16}$$

3 Calculation Results

As an example, let us consider the structural parameters of a dispersion unit, the prototype of which will be the DU of the H-50 cytophotometer (Ortho Instruments), calculated using the considered technique [3]. The diameter of ($\varepsilon = 1$) outlet thin opening of RFC7 is selected as $2r_0 = 80$ μm. The velocity of the jet flow is $V_j = 11.5$ m/s. Taking into account Eq. (16), the DU's resonant frequency must be $f_r = 32$ kHz. The parameters of the cylindrical ceramic PZT-19 are as follows [13]: $e_{32} = 7.2$ N/(m V), $c_1 = 3.3\cdot10^3$ m/s, $\rho_1 = 7\cdot10^3$ kg/m^3, $l_1 = 20$ mm; $r_2 = 11$ mm and $r_1 = 9$ mm. The cross-section of the quartz dish (9) ($\rho_9 = 2.2\cdot10^3$ kg/m^3, $c_9 = 5.93\cdot10^3$ m/s), 20 mm in height, is square with a side length of 0.2 mm. The remaining parts are manufactured from stainless steel ($c = 5.66\cdot10^3$ m/s, $\rho = 8.03\cdot10^3$ kg/m^3). The height and cross-sectional area of RFC2 are selected as $l_2 = 15$ mm, and $S_2 = 43.2$ mm^2. The length of RFC7 is $l_7 = 50$ mm; the condition of constant $z_{07} = 326$ kg/s is satisfied over all its length. A resonant frequency of DU of 32 kHz is provided thanks to the selection of ring 5 and 6 geometrical sizes (for example $l_5 = l_6 = 5.5$ mm; $S_5 = S_6 = 31.7$ mm^2) satisfying the frequency Eq. (5). From Eqs. (11) and (12) it follows that the selected parameters of DU for a resonant frequency of 32 kHz provide a velocity amplitude closed to maximum on the acoustic terminal: $v_r(0) = 0.95\,v_{r\,\max}(0)$. Concerning the flow of particles, we must consider the density of the liquid used (distilled water) and $T_t = 0.0727$ N/m. Taking into account Eq. (15), for $E = 20$ V the length of the jet's continuous section is 9-11 mm for $Q = 10\ -2$. It must be noted that the measured characteristics of the dispersion unit designed using the considered technique are in good accord with calculated data: for a resonant frequency of 32 kHz ($Q = 3$) the jet's monodispersion begins at a distance of 11 mm from the edge of the running fluid chamber when $E = 20$ V.

4 Conclusions

The purpose of this work was to develop an engineering technique for the simulation of the optimum design parameters of a jet dispersion unit for space sorting of particles in optical and electrical counters. This technique is based on solving a system of equations for the shift distribution in a cylindrical piezoelectric transducer and a running fluid chamber. Theoretical results obtained by calculation of the design parameters for an industrial dispersion unit used in the H-50 cytophotometer (Ortho Instruments) are in good agreement with the experimental results.

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6 Symbols and Abbreviations

A	force factor
c	velocity of sound
DU	dispersion unit
e	piezoelectric constant of voltage
E	electrical voltage
F	force
l	length
m	equivalent mass of the dispersion unit
M_i	mechanical mass of the ith element of the DU
r	radius
R	active mechanical resistance
RFC	running fluid chamber
S	cross-section
T	period of harmonic vibrations
v	vibrational velocity
V_j	jet velocity

z	impedance
β_i	$= 2\pi f l_i / c_i$ parameter of the ith element of the DU
ρ	material density
ω	circular frequency
ε	contraction factor of the opening

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