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Single-multiple scattering technique to simulate a time-dependent signal of a pulse laser radar with separately located illuminating and receiving platforms

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Abstract. A technique for calculating the time-dependent signal of a laser radar with a narrow angle of the field of view and separately located receiver and laser is proposed. The laser is assumed to have a short pulse and the results are valid for relatively uniform meteorological situations. This technique is based on the use of a single-multiple scattering model of the aerosol space and 'weighted' influence energetic coefficients of elementary volumes of aerosol space, taking account of spatial, energetic, temporary and other parameters of a radiation source and receiver. Taking into consideration a number of approximations, a relatively simple engineering technique to estimate a laser radar signal is described.

1. Introduction

Laser radars are based upon the principle of recording the radiation scattered by aerosols and they are widely used for the investigation (remote [1] and laboratory [2]) of aerosol parameters for process testing, for solving various problems in meteorology, and for the navigation of moving objects.

As an example, a naval navigation laser radar is presented in figure 1. Platform 2, on which a receiving system is located, has to move from the basic platform 1 towards a target. Because of the naval meteorological situation (for which the meteorological distance of visibility S_m is 1000–5000 m), the receiving system does not detect the target located in the far field at the first stage of the above movement. On the basic platform 1 with a pulsed laser, an operator having previous information about the target coordinates shows the 'laser road' toward the target. The receiving system, detecting the electromagnetic field scattered on the aerosol, moves towards the 'quasi target'. The quasi target is a part of scattered aerosol space identified by the receiver system, taking account of a given algorithm of a navigation program. For example, the quasi target can be identified as a part of aerosol space in the far field, from which the signal noise relationship is about one. Let the coordinate system XYZ of the illuminating system be the main coordinate system. As shown in figure 2, the propagation vector of the laser beam and the optical axis of the receiver system are located in the XOZ plane and intersect at an angle ψ. The contour *klmn* in the plane XOZ is formed by intersection of the laser beam with the receiver field of view. The location $z_c(t)$ of the centre of the illuminated zone depends on the time

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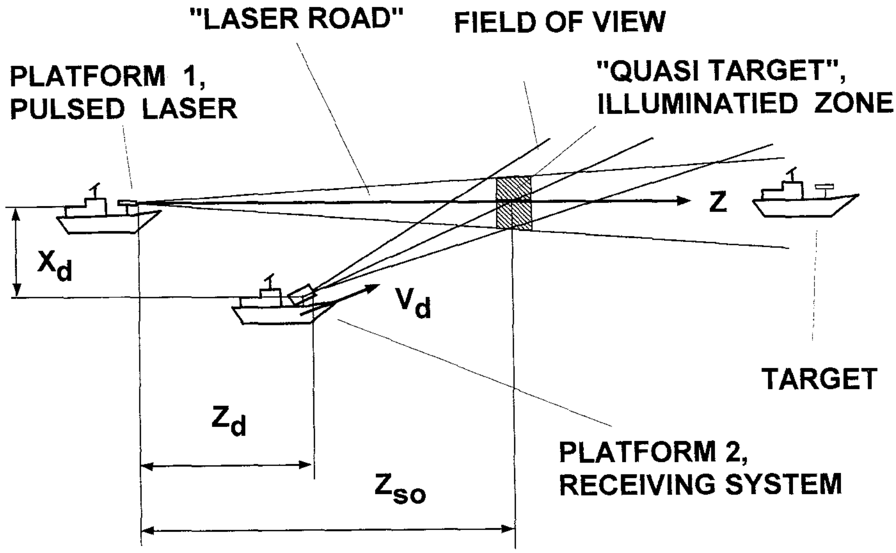

Figure 1. General scheme of a naval navigation system based on the laser radar principle.

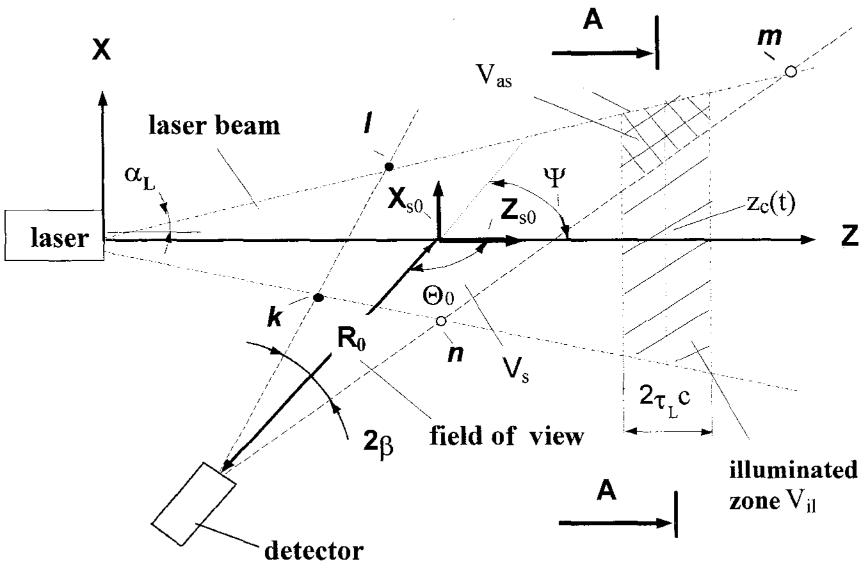

Figure 2. Laser radar system geometry.

of a single pulse measurement because the laser pulse propagates with the velocity of light c:

$$z_c(t) = [(tc)^2 - z_d^2 - x_d^2]/[2(tc - z_d)], \qquad (1)$$

where $x_d(t_r)$ and $z_d(t_r)$ are the coordinates of the detector at the time t_r, and t is the time elapsed since the beginning of propagation of the single illuminating pulse. When $t = 0$ the value of the radiation flux of this pulse reaches a maximum. Thus the scattering flux incident upon the detector is also time-dependent. The so-called time-dependent lidar equations for single [2], [3] and multiple [4] scattering aerosol models are well known.

Our aim is to develop a quasi-rigorous technique, based on the single-multiple scattering model of aerosol space, for calculating the time-dependent cross-section of the laser radar. The receiver is assumed to have a narrow field of view angle 2β and a short laser pulse of duration $2\tau_L$. By using two kind of approximations (the single scattering approximation described by Deirmendjian [8] and approximations of laser pulse propagation) we have developed a relatively simple engineering technique for estimating the radar signal. The purpose is to solve navigation problems of moving objects in the case of a relatively uniform meteorological situation.

2. Single-multiple scattering model of aerosol space

Figure 3 illustrates an interaction between a laser pulse and aerosol space. According to the above mentioned single-multiple scattering model, we can describe the aerosol space as a superposition of ith elementary scattering volumes $S_i(\mathbf{X}_i)$, characterized by position vectors $\mathbf{X}_i$ in the main XYZ coordinate system.

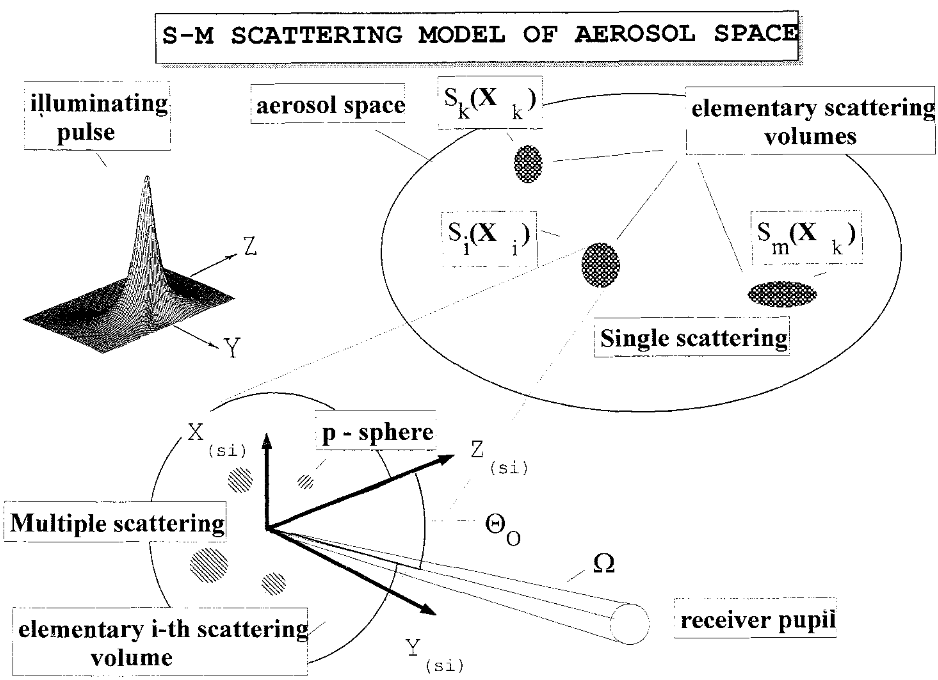

Figure 3. Single-multiple scattering model of aerosol space.

On the other hand, the ith elementary volume incorporates an ensemble of p spheres. The calculation of the volume differential cross-section $d\sigma_{vs}(\theta_i, \varphi_i, \mathbf{X}_i)$ of the scattering by the ith elementary volume, which is at a distance $z_s(\mathbf{X}_i)$ from the laser and at a distance of $R_s(\mathbf{X}_i)$ from the detector, should be realized in the spherical $(XYZ)_{si}$ coordinate system of the mentioned volume taking into account the *multi* scattering technique, created, for example, by Hamid [5].

It is evident that the following relationships are valid for the far field scattering and for a narrow angle of the receiver field of view (see figure 1).

$$\theta_i \simeq \theta, \ \varphi_i \simeq \varphi, \ R_i \simeq R, \ \mathbf{X}_i \simeq \mathbf{X}, \tag{2}$$

where $\{\theta, \varphi, R\}$ are the spherical coordinates in the main scattering coordinate system $(XYZ)_{s0}$, the centre of which is located at the intersection point of the propagation vector of the laser beam and the optical axis of the receiver system.

Hence, the volume cross-section $\sigma_{vs}(\theta_0, \varphi_0, \Omega, \mathbf{X})$ taken for scattering by the ith volume into the solid angle $\Omega(t)$ (defined by the receiver aperture) is equal to:

$$\sigma_{vs}(\theta_0, \varphi_0, \Omega(t), \mathbf{X}) = \int_{\Omega(t)} d\sigma_{vs}(\theta, \varphi, \mathbf{X})\, d\Omega, \quad [\mathrm{m}^2\,\mathrm{m}^{-3}], \tag{3}$$

where $d\Omega = \sin\theta\, d\varphi\, d\theta$ is the unit solid angle; $\{\theta_0, \varphi_0, R_0\}$ are the spherical coordinates of the receiver centre in the $(XYZ)_{s0}$ coordinate system of the scattering space V_s *and* $\theta_0 = \pi - \psi$.

In general a powerful pulsed laser is the radiation source. Owing to the pulsed nature of the radiation from the elementary scattering volumes of space V_s (which is common for the intersected laser beam and the field of view) the detected signal will have a time dependence. This can be estimated by the function $f_1(t_i)$ ('weighted' coefficient) which describes the form of the radiation pulse:

$$f_1(t_i) = \exp[-2t_i^2/\tau_L^2], \tag{4}$$

where τ_L is half of the full pulse duration at the $\exp(-2)$ level; t_i is the parameter which determines the influence of the elementary volume $S_i(\mathbf{X})$ on the receiving signal at the moment t:

$$t_i(\mathbf{X}, t) = t - [z_s(\mathbf{X}) + R_s(\mathbf{X})]/c. \tag{5}$$

Consequently the radiation flux density in any beam section at any instant t is calculated by:

$$\varphi(\mathbf{X}, t) = P_L(z) f_1(t_i) f_2(\mathbf{X}), \qquad [\mathrm{W\,m}^{-2}], \tag{6}$$

where $f_2(\mathbf{X})$ is a function taking into consideration the non-uniformity of the radiation distribution on the beam cross-section; $P_L(z)$ is the power of radiation on the axis Z. This function takes the following form for the TEM_{00} laser mode:

$$f_2(\mathbf{X}) = \frac{2}{\pi\omega_L^2(z)} \exp\left\{-2\frac{x^2+y^2}{\omega_L^2(z)}\right\}, \quad P_L(z) = \frac{2}{\sqrt{\pi}} \frac{Q_L(z=0)}{\tau_{L0}} \tau_t(z), \tag{7}$$

where

$$\omega_L(z) = \{[\omega_L(z=0) + z\tan\alpha_L]^2 + \omega_t^2(z)\}^{1/2}, \tag{8}$$

is the radius of the laser beam in the cross-section z; α_L is the beam divergence;

$\omega_t(z)$ is due to the increase of the laser beam because of atmospheric turbulence; $Q_L(z=0)$ is the energy of radiation in the cross-section $z=0$;

$$\tau_t(z) = \exp\left[-\int_0^z \beta_{ext}(z')\,dz' \right] \tag{9}$$

is the coefficient of atmosphere transmittance along the optical path with the length z; $\beta_{ext}(z')$ is the volume extinction coefficient which is related to the meteorological distance of visibility $S_m(z')$ by an empirical formula [6], for example:

$$\beta_{ext}(z') = \frac{3{\cdot}91}{S_m(z')}\left(\frac{0{\cdot}55}{\lambda}\right)^{0{\cdot}585\,S_m(z')^{3/2}}, \quad [m^2\,m^{-3}], \tag{10}$$

where z' is the coordinate characterizing the elementary optical path.

All elementary scattering volumes located at the moment t in the region V_s determine the total detected radiation flux, calculation of which can be simplified assuming *single*-scattering by these volumes:

$$P(t) = \int_{V_s(t)} \varphi(\mathbf{X}, t)\sigma_{vs}(\theta_0, \varphi_0, \Omega, \mathbf{X})\tau_t(R_s(\mathbf{X}))\,dV, \quad [W], \tag{11}$$

where $dV = dx\,dy\,dz$. It is evident that without significant decrease in the accuracy of calculation, the limits of the integration with respect to z can be narrowed down to the dimension commensurable with the duration of the incident pulse

$$z_c(t) - k\tau_L c \leqslant z \leqslant z_c(t) + k\tau_L c, \tag{12}$$

where k can take values between 1 and 4 depending on the accuracy of the calculation in this approximation.

3. Engineering technique to simulate the radar cross-section

Equation (11) allows us to carry out the rigorous calculations of the time-dependent signal for the short pulsed laser radar. To simplify the above simulation, the following assumptions can be made.

Let the laser pulse have a rectangular shape with duration $2\tau_L$ as described by

$$f_1(t_i) = \begin{cases} 1, & \text{for } -\tau_L \leq t_i \leq \tau_L, \\ 0, & \text{otherwise}, \end{cases} \qquad P_{Lr}(z, t) = \begin{cases} P_{Lr}(z), & \text{for } -\tau_L \leq t_i \leq \tau_L \\ 0, & \text{otherwise} \end{cases} \tag{13}$$

where

$$P_{Lr}(z) = \frac{Q_L(z)}{2\tau_L} = \frac{(2\pi)^{1/2}}{4} P_{L\,max}(z) \tag{14}$$

is the radiation flux in the rectangular pulse, resulting from the following normalization condition:

$$Q_L(z) = \int_{-\infty}^{\infty} P_L(z, t)\,dt_i = P_{L\,max}\frac{(2\pi)^{1/2}}{2}\tau_L = P_{Lr}2\tau_L. \tag{15}$$

Thus taking into account the above mentioned assumption, the distribution of the flux density in a cross-section z can be written;

$$\varphi_{a1}(\mathbf{X}, t) = \begin{cases} P_{Lr}(z)f_2(\mathbf{X}), & \text{for } z_c(t) - \tau_L c \leq z \leq z_c(t) + \tau_L c \\ 0, & \text{otherwise} \end{cases}. \tag{16}$$

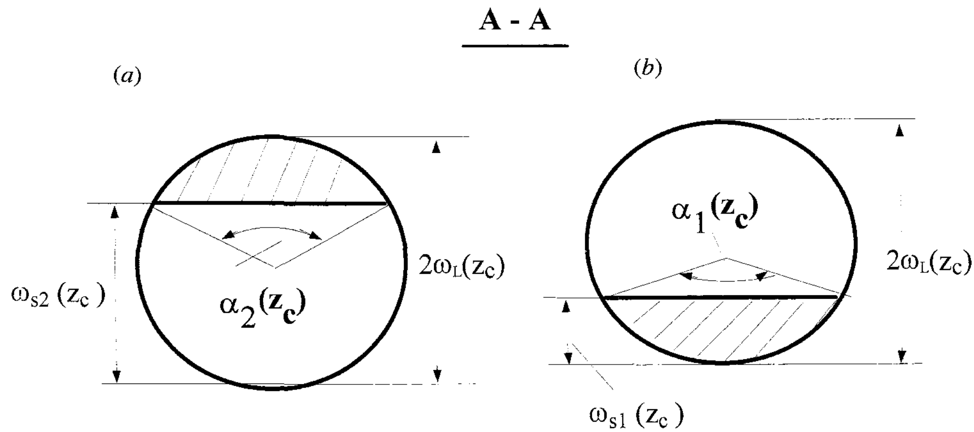

Figure 4. Simplified model of the laser beam cross-section: (*a*) $z_n < z_c < z_m$ ($j = 2$) and (*b*) $z_k < z_c < z_1$ ($j = 1$).

The incident flux density is written:

$$\varphi_{a2}(z, t) = \begin{cases} \dfrac{P_{Lr}(z)}{\pi\omega_L(z)^2}, & \text{for } z_c(t) - \tau_L c \leq z \leq z_c(t) + \tau_L c \\ 0, & \text{otherwise} \end{cases} . \tag{17}$$

For the far field scattering, the narrow field of view and for the small beam divergence the following relationships are valid for all elementary scattering volumes;

$$\sigma_{vs}(\theta_0, \varphi_0, \Omega(t), z) \approx d\sigma_{vs}(\theta_0, \varphi_0, z)\Omega(t), \qquad [\mathrm{m^2 m^{-3}}]$$

and

$$\Omega(t) = \frac{A_0}{R_c^2(t)}, \quad [\mathrm{sr}] \tag{18}$$

where A_0 is the area of the detector aperture.

Hence taking account of the above approximations, the formula to determine the total scattering flux detected by the radar receiver can be written by

$$P_{a2}(t) = \tau_t[R_c(t)]\varphi_{a2}[z_c(t)]\sigma_R[\theta_0, \varphi_0, \Omega(t), z_c(t)], \tag{19}$$

where

$$\sigma_R(\theta_0, \varphi_0, \Omega(t), z_c(t)) \approx d\sigma_{vs}(\theta_{0c}, \varphi_{0c})\Omega(t)V_{as}(t), \quad [\mathrm{m^2}] \tag{20}$$

is the time-dependent radar cross-section; $\{\theta_{0c}, \varphi_{0c}, R_c\}$ are the spherical coordinates of the receiver centre in the $(XYZ)_{0c}$ coordinate system of the illuminating zone;

$$R_c(t) = [(z_c(t) - z_d)^2 + x_d^2]^{1/2}: \tag{21}$$

$$\varphi_{a2}(z_c) = \frac{Q_L(z = 0)}{\pi\omega_L^2(z_c)2\tau_L}\tau_t(z_c); \tag{22}$$

$$V_{as}(t) = \int_{V_{as}} dV \approx V_{il}(t)k_c(t) \tag{23}$$

is the scattering volume in the case of the mentioned approximations, and

$$V_{il}(t) \approx 2\pi\omega_L^2(z_c)\tau_L c \tag{24}$$

is the scattering volume of the illuminated zone.

Taking into account a simplified model of the laser beam cross-section presented in figure 4, the cutting coefficient of the field of view can be presented for the far field by the following equation:

$$k_c(t) = \frac{V_{as}(t)}{V_{il}(t)} \approx \begin{cases} 0, & \text{for } z_c \leq z_k, \quad \text{or } z_c \geq z_m; \\ 1, & \text{for } z_1 \leq z_c \leq z_n; \\ k_{c1}(z_c), & \text{for } z_c \leq z_{min}, \quad \text{where } z_{min} = \min(z_l; z_n); \\ 1 - k_{c2}(z_c), & \text{for } z_{max} \leq z_c \leq z_m, \quad \text{where } z_{max} = \max(z_l; z_n); \\ k_{cl}(z_c) - k_{c2}(z_c), & \text{for } z_n \leq z_c \leq z_l; \end{cases} \tag{25}$$

where

$$k_{cj}(z_c) = \frac{S_{cj}(z_c)}{\pi\omega_L^2(z_c)}; \quad S_{cj}(z_c) = \frac{\omega_L^2(z_c)}{2}(\alpha_j(z_c) - \sin\alpha_j(z_c)) \tag{26}$$

is the square of the segment characterized by the angle $\alpha_i(z_c)$ in the cross-section z_c [7]; $j = 1, 2$;

$$\alpha_j(z_c) = \begin{cases} 2\arccos\dfrac{\omega_L(z_c) - \omega_{sj}(z_c)}{\omega_L(z_c)}, & \text{for } z_k \leq z_c \leq z_l \quad \text{and } j = 1; \\ & \text{or } z_n \leq z_c \leq z_m \quad \text{and } j = 2; \\ 0, & \text{otherwise}; \end{cases} \tag{27}$$

$$\omega_{s1}(z_c) = \begin{cases} (z_c - z_k)[\tan(\psi + \beta) + \tan\alpha_L], & \text{for } z_k \leq z_c \leq z_l; \\ 2\omega_L(z_c), & z_c > z_l; \\ 0, & \text{otherwise}; \end{cases}$$

$$\omega_{s2}(z_c) = \begin{cases} (z_c - z_m)[\tan(\psi - \beta_L) + \tan\alpha_L], & \text{for } z_n \leq z_c \leq z_m, \\ 2\omega_L(z_c), & z_c > z_m, \\ 0, & \text{otherwise}, \end{cases} \tag{28}$$

where $\omega_{sj}(z_c)$ is the part of the laser beam diameter $2\omega_L(z_c)$ in the cross-section z_c which is located under the surface of the cone of the field of view [7]. Ignoring atmospheric turbulence, the coordinates of the contour *klmn* (see figure 2) can be estimated by

$$z_k = \frac{x_d - \omega_{L0} + \mu_1}{(\mu_1/z_d) + \tan\alpha_L}, \quad z_l = \frac{x_d + \omega_{L0} + \mu_1}{(\mu_1/z_d) - \tan\alpha_L},$$

$$z_n = \frac{x_d - \omega_{L0} + \mu_2}{(\mu_2/z_d) + \tan\alpha_L}, \quad z_m = \frac{x_d + \omega_{L0} + \mu_2}{(\mu_2/z_d) - \tan\alpha_L}, \tag{29}$$

where

$$\mu_1 = z_d\tan(\psi + \beta), \ \mu_2 = z_d\tan(\psi - \beta), \ \omega_{L0} = \omega_L(z = 0). \tag{30}$$

Thus taking into account the above mentioned approximations, the formula to determine the detected scattering flux can be written by

$$P_{a2}(t) = K_{slow}(t)K_{fast}(t), \quad [\mathrm{W}] \tag{31}$$

where

$$K_{slow}(t) = Q_L(z=0)A_0 \mathrm{d}\sigma_{vs}(\theta_{0c}(t), z_c(t))c, \quad [\mathrm{W\,m^2}] \tag{32}$$

is the slow-time-dependent coefficient (for uniform naval meteorological situation this coefficient is constant, because $\mathrm{d}\sigma_{vs}$ is approximately constant between the basic platform 1 and the target);

$$K_{fast}(t) = \tau_t\{z_c(t) + R_c(t)\}k_c(t)/[4\pi R_c^2(t)], \qquad [\mathrm{m^{-2}}] \tag{33}$$

is the fast-time-dependent coefficient defined, in general, by the relationship between the space duration of the laser pulse and the geometry of the common area of the field of view and the contour of beam (or in the other words by the coefficient k_c).

4. Single-single scattering model of aerosol space

The calculation of the volume cross-section of multiple scattering $\mathrm{d}\sigma_{vs}(\theta, \varphi, z)$ is the most complicated [5]. However the simulation is simplified when single scattering [8] by spherical aerosol particles can be adopted. It is known that single scattering is valid when the optical depth is smaller than two [9]. Analysis of figure 1 shows that because the laser beam propagates along the surface of the sea, where meteorological situation is relatively uniform, the above condition is often satisfied and calculations are simplified:

$$\mathrm{d}\sigma_{vs}(\theta, z) = [\beta_s(z)/4\pi]\{\tfrac{1}{2}[P_1(\theta, z) + P_2(\theta, z)]\}, \tag{34}$$

where

$$\beta_\xi(z) = \frac{\pi}{k^3}\int_0^\infty \rho^2 n(\rho, z)k_\xi(m, \rho)\,\mathrm{d}\rho; \tag{35}$$

$$k_s = \frac{2}{\rho^2}\sum_{n=1}^{\infty}(2n+1)(|\alpha_n(m, \rho)|^2 + |\beta_n(m, \rho)|^2), \tag{36}$$

$$k_{ext}(m, \rho) = \frac{2}{\rho^2}\sum_{n=1}^{\infty}(2n+1)\,\mathrm{Re}\,\{\alpha_n(m, \rho) + \beta_n(m, \rho)\} \tag{37}$$

are the efficiency factors of scattering and extinction [10]; $\alpha_n(m, \rho)$, $\beta_n(m, \rho)$ are the complex coefficients which are calculated according to the known formulas of Mie theory [10]; $m = n - i\chi$ is the complex refractive index of the aerosol at the wavelength λ of incident radiation; $n(\rho)$ $[\mathrm{m^{-3}\,m^{-1}}]$ is the density of particle concentration distribution along the diffraction parameter $\rho = kr$; $k = 2\pi/\lambda$ is the wavenumber; r is the radius of the aerosol particle; ξ is the subscript denoting the scattering ($\xi = \mathrm{s}$) volume coefficient β_s or the extinction ($\xi = \mathrm{ext}$) volume coefficient β_{ext}; $P_\zeta(\theta, z)$ are the normalized coefficients of the scattering matrix for an ensemble of polydispersion particles:

$$P_\zeta(\theta, z) = \frac{4\pi}{k^3\beta_s(z)}\int_0^\infty n(\rho, z)|S_\zeta(m, \rho, \theta)|^2\,\mathrm{d}\rho, \tag{38}$$

where $S_\zeta(m, \rho, \theta)$ are the elements of the scattering matrix for a single particle [10];

$\zeta = 1,2,3,4$ ($\zeta = 1,2$ for a spherical particle). In the case of single scattering model of aerosol the first coefficient is simplified:

$$K_{slow}(t) = Q_L(z=0)A_0\beta_{ext}(z_c(t))\omega_\alpha(z_c(t))\frac{P_1(\theta_{0c}(t)) + P_2(\theta_{0c}(t))}{2}c, \quad [\mathrm{Wm}^2] \qquad (39)$$

where $\omega_\alpha(z) = \beta_s/\beta_{ext}(z)$ is the albedo of a scattering aerosol [3]. It is well known that for backscatter ($\theta \approx 180°$) the following relationship is valid: $P_1(\theta) \approx P_2(\theta)$.

5. Calculation results

As an example, we have simulated the signal of a navigation laser radar working in a quasi uniform naval meteorological situation characterized by the meteorological distance of visibility $S_m = 100$ m, for which the single–single scattering model is valid [9]. Parameters of the pulsed laser ($2\tau_L = 10^{-8}$ s; $Q_L = 0{\cdot}3$ W s; $\lambda = 1{\cdot}06$ [µm]; $\alpha_L = 1$ mrad) were chosen taking into account of naval navigation tasks for the far field location of the target. The distance between the target and the basic platform 1 is more then 20 000 m (see figure 1). Threshold irradiance of the detector is $E_t = 10^{-12}\,\mathrm{W\,m^{-2}}$ and the coordinates of the receiver system (platform 2) at a given real time t_r are: $x_d = 20$ m; $z_d = 4000$ m. The single-single scattering model of aerosol and table data of coefficients $P_1(\theta)$ and $P_2(\theta)$, presented by Deirmendjian [8], were used. The signal-noise relationship (SNR) is calculated by formula

$$SNR = \frac{E(t)}{E_t}, \qquad (40)$$

where $E(t) = P(t)/A_0$ is the irradiance of the receiver pupil, and is presented for the following coordinates of the intersection point: $z_{so} = 4300$ m (figure 5(*a*)); $z_{so} = 5000$ m (figure 5(*b*)) and $z_{so} = 6000$ m (figure 5(*c*)). The shape of signals is defined by the fast time-dependent coefficient (33) only, which depends in general on the relationship between the space duration of laser pulse $2\tau_L c$ and the geometry of the common area *klmn* (see figure 2) between the field of view and the contour of beam. The coefficient K_{slow} (32) is a constant because of the uniform meteorological situation.

The results obtained are used to optimise a navigating program of moving receiver and laser platforms. For example, for the given position of the receiver platform $z_d = 4000$ m, it should be rotated to the optimal intersection point (quasi target, see figure 1) characterized by the coordinate z_{s0}, approximately 6000–6500 m, for which the above mentioned optimization criterion needs to be fulfilled (or in the other words for which the SNR is about 1–4·5).

By analogy with the single scattering bank published by Deirmendjian [8], it is a good idea to create a data bank of volume cross-section $\sigma_{vs}(\theta, \phi)$ for different kinds of aerosol taking into account of multiple scattering [5]. These data allows us to simulate the signal of laser radar by using a more exact equation (11).

6. Conclusion

The technique for calculating the time-dependent radar signal (11) was derived with the aims of increasing the design efficiency of laser radar with arbitrary configuration and of decreasing the expenditure required for this work. This technique is based on the use of a single-multiple model of aerosol space, ‘weighted’ influence energetic coefficients of elementary volumes of aerosol space taking into account spatial, energetic, temporary and other parameters of the radiation source

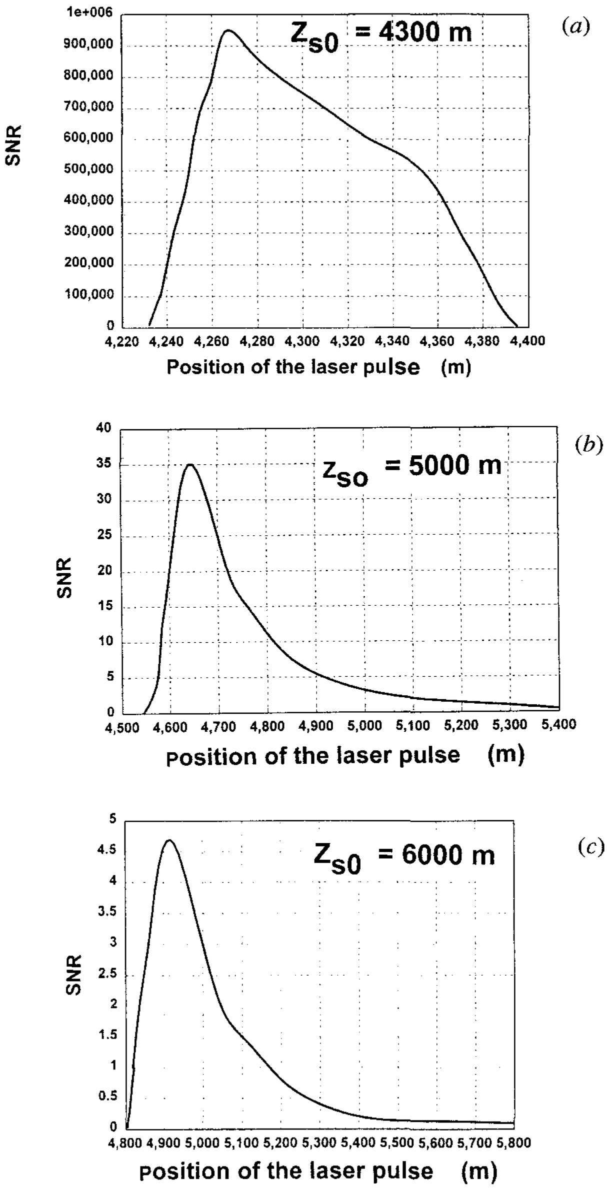

Figure 5. Signal-noise relationship $SNR = E(t)/E_t$ of a naval navigation laser radar as a function of a pulse position, is presented for the following coordinates of the intersection point between the laser propagation vector and the optical axis of the receiver: (*a*) $z_{so} = 4300$ m; (*b*) $z_{so} = 5000$ m and (*c*) $z_{so} = 5000$ m.

and receiver. Taking into consideration a single–single scattering approximation and a number of approximations of illuminating pulses, the relatively simple formulae for the time-dependent radar cross-section (20) and the detected scattering flux (31), (35) are obtained. This technique is scheduled for integration into the newly created program [11].

It is possible to use the technique for simulation of the volume cross-section of navigation and atmosphere sounding laser radars for more complicated meteorological situations.

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